Optimal Capacity Assignment in IP Networks

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***Abstract*—We present an analysis of the problem of routing and bandwidth allocation problem in packet switched networks. In this problem, we identify a route for every pair of communicating nodes and then assign a capacity to each link in the network in order to minimize the total line capacity and delay costs. We have developed a mathematical programming formulation which is an efficient solution. This formulation is indicated to be effective procedure based on computational results across a variety of networks.**

***Index Terms*—Optimization routing, capacity assignment, network delay, link flows, mathematical programming solution**

I. Introduction

The capacity and flow assignment (CFA) problem [1] involves determining the routing between origin-destination (O-D) pairs and assigning capacities to the links used by these routes; this is a dilemma in the design of the packet switched networks. One of the objectives in network design is to minimize total system costs, which consist of a) connection costs depending on capacities and end-to-end delay, and b) transfer costs which are incurred do to the limited line and node capacities. A favorable design will make simultaneous decisions on both routing and link capacities as they are closely related. In a recent paper [2] the authors developed and presented an efficient algorithm for MPLS network optimization subject to constraints on routing imposed by QoS and other considerations. The result of the experimentation was to verify how to optimize the use of the available bandwidth and minimize the effects of network congestion with MPLS TE.

In this paper, we focus on the question of link capacity assignment: what capacity throughput should be made available on each link of the network to provide a specified grade of service (network performance)?

Topological design of distributed computer networks in capacity assignment is a subject with comparatively few published results, given that it has been studied since the late 1970's. (See examples [6], [7] and [11]). As such, this complex field continues to be challenging to study; however, there are some recent findings on relative bandwidth allocation techniques, (See examples [8], [3], [4], and [5]).

Most of the published research devoted to this problem handles capacity assignment and flow assignment separately. All these research approaches to capacity assignment, also utilize different performance criteria. In the capacity assignment problem [6], [7], [8] the routing policy is assumed to be given and the best capacity for each link is sought among a set of discrete set of line capacities. The flow assignment problem [9], [10], [11] starts from a given assignment of link capacities, primary routes between O-D pairs are determined to minimize either the average message delay or the maximum message delay in the network.

An additional approach is to improve an existing network by redistributing the link capacities while maintaining the total sum of all capacities of the network.

Kleinrock [11] notes that the selection of an appropriate algorithm to allocate capacities will depend on the cost-capacity structure, on the presence of additional topological constraints, on the degree of human interaction allowed and, finally, on the tradeoff between cost and precision required by the particular application. Kershenbaum describes a capacity assignment approach in [10] that guarantees an optimal solution but can take an inordinate amount of time. The algorithm can yield some approximate results by alternatively strengthening the dominance criterion. Another approach originally proposed by Whitney [12] guarantees a solution in a very reasonable amount of time and also gives a bound on the quality of the solution it obtains (which is not, in general, optimal).

Balakrishnan and Graves [13] studied the special case of piecewise linear costs in directed networks where each link is assigned a capacity and a path is identified for each O-D pair. They formulated the problem as a mixed integer program and developed a composite algorithm to generate both lower bounds and feasible solutions. The model however does not take into account the delay issue that arises when link capacity utilization reaches certain levels.

In [14] Fratta et al. incorporate heuristic methods for capacity assignment developed in [7] into a more general procedure that iterates between a composite capacity assignment algorithm and flow assignment phase until a local optimum is reached. They also describe a priority assignment scheme which, with high likelihood, yields a less costly capacity assignment satisfying the delay requirements. A similar iterative procedure which alternates between capacity and flow assignment is used by Gerla and Kleinrock [15] where different heuristic methods based on the flow deviation algorithm [14] for static route assignment is presented.

Gavish and Neuman [18] introduced a nonlinear integer programming formulation of the problem which minimizes the total system cost while selecting a route for each origin-destination pair and assigning a capacity to each link. One of the major weaknesses of this procedure is that a priori given set of possible routes are used throughout the solution procedure. Computational tests in Gavish and Hantler [21] have revealed that a poor choice of candidate routes can lead to poor solutions

The authors in [1] extended the work in [18] by considering all possible routes for every communicating node pair. They formulated the problem and used Lagrangean relaxation embedded in a subgradient optimization procedure to obtain lower bounds as well as feasible solutions to the problem. They included cut constraints which are redundant in the original problem to improve the lower bounds. These cut constraints are assumed to be defined before the solution procedure starts. Obviously the quality of the solutions depends heavily on the number and choice of the cuts.

The paper is structured as follows. We first describe our modeling framework then a nonlinear integer programming formulation of the network design problem is given in Section II. The following Section presents our optimal capacity assignment algorithm. Computational experiments performed on our solution procedure are reported in the subsequent sections.

II. Modeling and Analysis

In this section we introduce the appropriate notation and definitions and follow this with the collection of assumptions that define our model of the network. We identify the class of analysis and synthesis problems that confront us in a network studies.

Consider a physical network consisting of a set of N nodes denoted by $\mathcal{N}$ and a set of L physical links denoted by $\mathcal{L}$. the nodes represent the routers in IP network. The traffic requirements are specified by an $N \times N$ matrix $\mathcal{R}_e = r_{ij}$, called the requirement matrix, whose entries are non-negative. Let C_{ij} denote the capacity in bandwidth units of the physical link from an origin node i to a destination node j. The set of routes connecting O-D pair (o, d) is denoted by $\mathcal{R}_{o,d}$. Each route consists of a non-cycling sequence of physical links.

In the network design problem, the messages are offered to O-D pair (i, j) according to a Poisson process with mean rate λ_{ij}. The average message length from node i to node j is exponentially distributed with mean $1/\mu_{ij}$. Let $\rho_{ij} = \lambda_{ij}/\mu_{ij}$ denote the intensity of the offered traffic stream. Let f_r denote the flow on route r. The total flow F_{ij} on link (i, j) is denoted by

$$F_{ij} = \sum_{r \in \mathcal{A}_{ij}} f_r$$

where $\mathcal{A}_{ij}$ is the set of routes that use link (i, j).

We are interested in the numerical solution of the following network flow problem, subject to a constraint that the total link capacity not exceed C_{ij}:

Minimize: The average end to end network delay.

$$T = \sum_{(i,j)} \frac{F_{ij}}{\gamma} T_{ij} \tag{1}$$

where $\gamma = \sum_{(i,j)} \lambda_{ij}$ is the total message arrival rate from external sources (bits/sec) and T_{ij} is the average delay experienced by a message on link (i, j) (sec) *subject to:*

$$0 \leq F_{ij} \leq C_{ij} \qquad \forall i, j \in \mathcal{N} \tag{2}$$

We further assume that the cost of constructing the channel with capacity C_{ij} is given by $d_{ij}C_{ij}$, an arbitrary function of the capacity and of the channel. We let D represent the cost of the entire network, which we assume to consist only of the cost for channel construction, and so we have

$$D = \sum_{(i,j)} d_{ij} C_{ij} \tag{3}$$

where d_{ij} is the positive cost per unit capacity on link (i, j).

We have earlier defined the message delay as the total time that a message spends in the network. Of most interest is the *average* message delay (1) and we take this to be our basic performance measure.

In any practical network design procedure, a large number of design variables suggest themselves. Among these we include: the selection of channel capacities; the form of routing procedure; the form of flow control procedure; the topological design of the network; the storage capacity at each node; the choice of hardware and software programs to be used for the switching computer; the partitioning of messages into various-size packets; and so on. Since we are interested mainly in the queueing marvels in this paper, we discuss neither the hardware nor many aspects of the software design of the switching computer itself any further.

III. CA Problem Formulation

This section begins by considering the problem of assigning optimal capacities to the links in the network given the link topology (location of the nodes and links) and link flows. The traffic requirements between the node pairs are measured in bits per second. We assume a flow distribution – a flow on each link which satisfies the requirements.

The objective is to compute the optimal link capacities for a network where the topology and traffic flows are known and fixed which minimizes the average network delay subject to the linear overall cost of the system:

$$\sum_{(i,j)} d_{ij} C_{ij} \tag{4}$$

In essence, we want to establish both the scope of resource capacity required for the given demand volume, and how to reasonably, efficiently distribute it in the network under a set of routing/flow constraints. This determination, which is typically found in medium to long-term network planning, is broadly known as uncapacitated design.

The uncapacitated problem has been studied by, among others Rohne et al [16] who first design a physical network under the assumption that all traffic will be routed on a node-by-node basis, and then configure the virtual circuit highways by cross connecting flows in the physical network according to a cost model. Bauschert [17] also considers the uncapacitated problem and uses a set of iteration loops to simultaneously design a packet switched network and virtual path routing policy. This scheme relies on an initial pre-selection of paths and includes linear programming models.

Once the capacity in a network is known and the demand volume is given, the problem changes to how to allocate flows on different paths is a manner that optimizes a given network goal (e.g., minimum cost routing or maximum total revenue). The system costs are composed of connection costs which depend on link capacities and delay costs incurred by users due to the limited capacities of the links and the resulting queueing at intermediate nodes.

We wish to focus on three very basic design parameters that we must consider: first is the selection of the channel capacities C_{ij}; second is the selection of the channel flows F_{ij}; and third there is the topology itself. All of these may be varied to improve network performance. The notion of "optimum design" is extremely difficult to achieve in any realistic network design; however we define, the performance criterion, the average message delay T, and attempt to minimize this quantity (thereby optimizing performance). This approach will allow us to make some important qualitative statements about network design and performance. Of course, any optimization problem must be subject to some form of cost constraint, and here we choose the fixed cost constraint given in Eq. (3). Therefore we have a performance measure T, a cost constraint D, and three variables design "parameters," C_{ij}, F_{ij}, and the topology.

This study overcomes thoughtful shortcoming of previous methods suggested in past research. In the routing selection process, these methods assume that a set of pre-specified candidate routes chosen from among all possible routes is given for every communicating O-D pair. Obviously, the quality of the solutions obtained by these methods depends heavily on the choice of the candidate route sets generated before the procedure is applied. The use of only a subset of all possible routes by these methods results in a practical limitation which is the possibility of generating lower bounds higher than the values of the optimal solutions to the routing and capacity assignment problem. Our solution method eliminates this limitation by considering all possible routes for every communicating node pair.

One of the primary attractions of the technique known as Lagrangean relaxation is that it provides both upper and lower bounds on the value of the objective function [19]. That is, we know the optimal objective function value lies between the value of the best feasible solution found and a value that it can be no better than. In this paper, a Lagrangean problem is formed by multiplying some of the constraints by Lagrange multipliers and adding them to the objective function. As a result, the Lagrangean problem is separable to a routing subproblem and a link subproblem. Each type of subproblem is further separable into subproblems for each link and for each communicating pair. The link subproblem consists of assigning a capacity to a link and the route subproblem deals with choosing a route for a communicating O-D pair.

To minimize the objective function, we proceed by using a Lagrange multiplier β and by forming the Lagrangean relaxation function as follows:

$$L = T + \beta\left(\sum_{(i,j)} d_{ij}C_{ij} - D\right) \tag{5}$$

where D is the total cost of the network and T is given by the $M/M/1$ delay function:

$$T = \frac{1}{\gamma}\sum_{(i,j)}\left(\frac{F_{ij}}{C_{ij} - F_{ij}}\right)$$

In Eq. (5), if we find the minimum value of L with respect to the capacity assignment, then we will have found the solution to the capacity assignment problem since the bracketed term is identically equal to zero. The parameter β is the undetermined multiplier to be evaluated.

If β is large enough, it is a penalty for violating the constraint on total capacity. If the sum of the C_{ij} exceeds D, the term in the brackets is positive and if multiplied by β increases the value of the objective to be minimized. Values of C_{ij} are sought which do not violate the constraint. If β is too large, however, it is possible to make this new objective function smaller by letting the sum of the C_{ij} become strictly less than D, thus minimizing the new objective function but not the original one. As β increases from zero, the sum of the C_{ij} decreases as the first term is traded in the objective against the second. There is a unique value of β which makes the sum exactly equal to D. This is the value sought along with the corresponding values of the C_{ij}.

As is usual in Lagrangean optimization problems, we set the partial derivatives $\partial L/\partial C_{ij}$ to zero:

$$\frac{\partial L}{\partial C_{ij}} = \beta d_{ij} - \frac{F_{ij}}{\gamma(C_{ij} - F_{ij})^2} = 0 \tag{6}$$

Solving for C_{ij} gives:

$$C_{ij} = F_{ij} + \frac{1}{\sqrt{\beta\gamma}}\sqrt{\frac{F_{ij}}{d_{ij}}} \tag{7}$$

The objective now is to find the value of β. Once we have evaluated the constant β, this will be our solution.

$$\sum_{(i,j)} d_{ij}C_{ij} = \sum_{(i,j)}\left(F_{ij}d_{ij} + \frac{1}{\sqrt{\beta\gamma}}\sqrt{F_{ij}d_{ij}}\right)$$

From this equation, solving for β gives,

$$\frac{1}{\sqrt{\beta\gamma}} = \frac{D - \sum_{(i,j)} F_{ij} d_{ij}}{\sum_{(i,j)} \sqrt{F_{ij} d_{ij}}}$$

Using this last form in the Eq. (7), the optimal solution to the capacity assignment problem is

$$C_{ij} = F_{ij} + \frac{D - \sum_{(i,j)} F_{ij} d_{ij}}{\sum_{(i,j)} \sqrt{F_{ij} d_{ij}}} \sqrt{\frac{F_{ij}}{d_{ij}}} \tag{8}$$

The algorithm assumes:

1) The nodes of the network and the input traffic flow for each pair of nodes are known.
2) A routing model determines the optimal flows F_{ij} of all links (i, j) given the link original capacities C_{ij}. We assume that the link flows minimize a cost function $\sum_{ij} D_{ij}(F_{ij})$ and F_{ij} can be determined by minimizing the average packet delay,

$$T = \frac{1}{\gamma} \sum_{(i,j)} \left(\frac{F_{ij}}{C_{ij} - F_{ij}} + F_{ij} p'_i \right)$$

based on the $M/M/1$ formula, where γ is the total input traffic into the network, and C_{ij} and p'_i are the capacity and the processing and propagation delay, respectively, of link (i, j). The algorithms described in [2] can be used for this purpose.

IV. The algorithm

This section describes the different steps of the capacity assignment algorithm and how it works.

The capacity assignment algorithm

Step 1: Select a network topology with initial capacities and requirements.

Step 2: Compute optimal link flows that minimize the average delay for the network using the FOA algorithm.

Step 3: Allocate the link capacities to minimize the delay with the link flows computed in step 2, given the constraints on the total cost of a system.

Step 4: Use these link capacities instead of the original capacities with the original requirements and go to step 2. The delay calculated in this step will be less than in step 2.

Step 5: Reallocate the optimal link capacities with the optimal link flows from step 4. The new delay will be smaller than the delay in step 3.

The iteration is repeated until the new network delay is not significantly smaller than the old delay. The convergence of the algorithm is guaranteed by the fact that there are only a finite (albeit large) number of shortest route flows, and repetitions of the same flow are not possible, as the delay is decreasing.

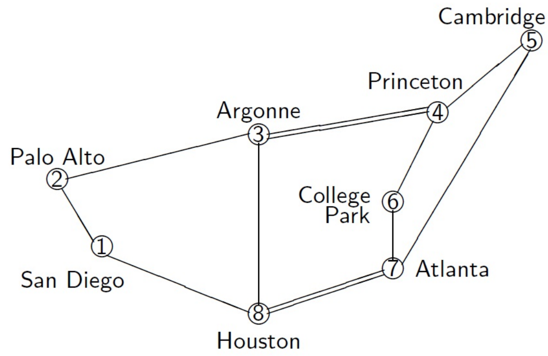

Fig. 1. An ATM infrastruture network

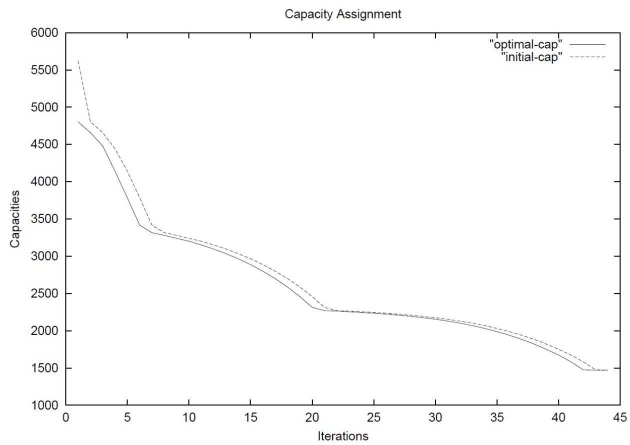

Fig. 2. The optimal versus the intial link capacities

In general, this approach only leads to a local minimum, not a global optimum. However, for the special case (i.e., $d_{ij} C_{ij} = d_{ij} C_{ij}^{\alpha} + d_{ij0}$ where d_{ij0} is a positive start-up cost, and $0 \le \alpha \le 1$), it is still possible to find a global minimum [19].

V. Network Performance Analysis

An application of our algorithm to the network topology shown in Fig. 1 is next introduced. The network model consists of 8-nodes and 10-links introduced and presented in [20] to compute the optimal link capacities. Each link carries traffic in both directions. The double lines between nodes 3 and 4 and 7 and 8 indicate that there are two links in each direction connecting these nodes. The capacity of each link is 5624 units.

We now compare the network optimal link capacities with their initial values. Table I shows the optimal link flows and capacities computed after the convergence of the capacity assignment algorithm. The optimal and initial link capacities' values calculated are displayed in Fig. 2

Figure 3 plots the capacity assigment for link (1-2) as the algorithm executes while the network delay is plotted in Figure 4 as the algorithm executes. The delays converge after 44

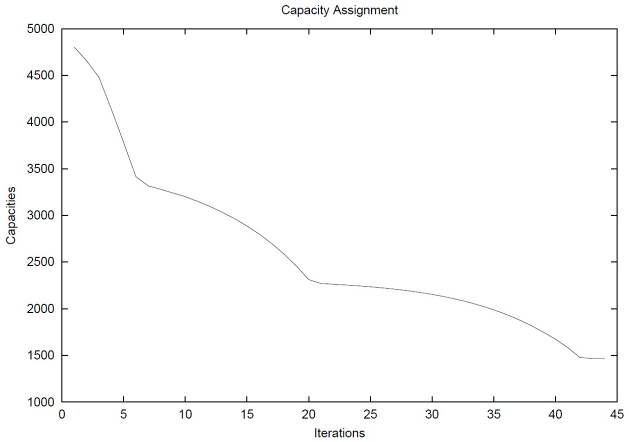

Fig. 3. Capacity assignment algorithm

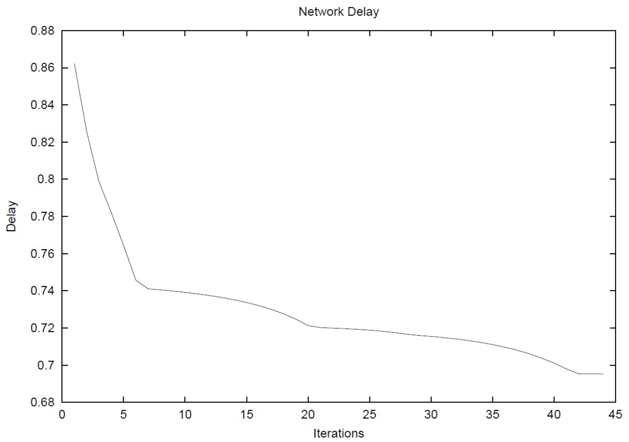

Fig. 4. Convergence of the algorithm

iterations. That is, when the newly calculated network delay is not significantly better than the previous one. Comparison with Figure 3 shows a strong correlation between the delay and the assigned capacity. This can be expected since the delay is a function of capacity. The delays decrease since the flows are shifted onto optimal paths in order to reduce the delays on congested links. This smaller flow on the congested links in turn leads to a decrease in the capacity required to achieve a given delay.

VI. Conclusions and Furure Work

In this paper, the routing and capacity assignment problem have been investigated. A mathematical programming formulation of the problem is presented and efficient solution procedures based on a Lagrangean relaxation of the problem are developed. Overall system costs are minimized by trading off link capacity costs versus expected network delay costs. The major incentive of the work here is that by using a route generation procedure as part of the Lagrangean procedure, routes are generated only when needed as an integral part of the solution method, resulting in reduced running times and storage requirements. In future work, we plan to develop and evaluate the extension of this algorithm from single- to multi-service network traffic.

TABLE I
Optimal link flows and link capacities

Links	Optimal Flows	Optimal Cap	Initial Cap
(1,2)	520	1469.4	5624
(1,8)	2280	4268.1	5624
(2,3)	7040	10533.4	5624
(3,4)	8800	12705.8	11248
(3,8)	3240	5609.9	5624
(4,6)	4480	7266.8	5624
(5,7)	2640	4779.3	5624
(6,7)	1200	2642.3	5624
(7,5)	2640	4779.3	5624
(7,8)	4040	6686.4	11248

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